UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent — B1
Solve a 3-term quadratic for e^x or for u — M1
Obtain simplified solution $e^x = 3$ or $u = 3$ — A1
Obtain final answer $x = 1.10$ and no other — A1 [4]

2 *EITHER*: Use chain rule — M1
obtain $\dfrac{dx}{dt} = 6\sin t\cos t$, or equivalent — A1
obtain $\dfrac{dy}{dt} = -6\cos^2 t\sin t$, or equivalent — A1
Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$ — M1
Obtain final answer $\dfrac{dy}{dx} = -\cos t$ — A1

OR: Express y in terms of x and use chain rule — M1
Obtain $\dfrac{dy}{dx} = k(2 - \dfrac{x}{3})^{\frac{1}{2}}$, or equivalent — A1
Obtain $\dfrac{dy}{dx} = -(2 - \dfrac{x}{3})^{\frac{1}{2}}$, or equivalent — A1
Express derivative in terms of t — M1
Obtain final answer $\dfrac{dy}{dx} = -\cos t$ — A1 [5]

3 **(i)** *EITHER*: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ — M1
Obtain quotient $x^2 + 4x + 3$ — A1
Equate remainder of form lx to zero and solve for a, or equivalent — M1
Obtain answer $a = 1$ — A1

OR: Substitute a complex zero of $x^2 - x + 1$ in p(x) and equate to zero — M1
Obtain a correct equation in a in any unsimplified form — A1
Expand terms, use $i^2 = -1$ and solve for a — M1
Obtain answer $a = 1$ — A1 [4]

[SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B. The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.]

(ii) State answer, e.g. $x = -3$ — B1
State answer, e.g. $x = -1$ and no others — B1 [2]

4 Separate variables and attempt integration of at least one side — M1
Obtain term $\ln(x + 1)$ — A1
Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ — M1
Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ — A1
Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ — M1
Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2}\ln\sin 2\theta - \frac{1}{2}\ln\frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm\frac{1}{2}$) — A1√
Rearrange and obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent — A1 [7]

5	**(i)**	Make recognisable sketch of a relevant graph over the given interval	B1	
		Sketch the other relevant graph and justify the given statement	B1	[2]
	(ii)	Consider the sign of sec $x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Convert the given equation to sec $x = 3 - \frac{1}{2}x^2$ or work *vice versa*	B1	[1]
	(iv)	Use a correct iterative formula correctly at least once	M1	
		Obtain final answer 1.13	A1	
		Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change in the interval (1.125, 1.135)	A1	[3]
		[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]		
6	**(i)**	State or imply $R = \sqrt{10}$	B1	
		Use trig formulae to find α	M1	
		Obtain $\alpha = 71.57°$ with no errors seen	A1	[3]
		[Do not allow radians in this part. If the only trig error is a sign error in $\cos(x - \alpha)$ give M1A0]		
	(ii)	Evaluate $\cos^{-1}(2/\sqrt{10})$ correctly to at least 1 d.p. (50.7684…°) (Allow 50.7° here)	B1√	
		Carry out an appropriate method to find a value of 2θ in $0° < 2\theta < 180°$	M1	
		Obtain an answer for θ in the given range, e.g. $\theta = 61.2°$	A1	
		Use an appropriate method to find another value of 2θ in the above range	M1	
		Obtain second angle, e.g. $\theta = 10.4°$, and no others in the given range	A1	[5]
		[Ignore answers outside the given range.]		
		[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]		
		[SR: The use of correct trig formulae to obtain a 3-term quadratic in tan θ, sin 2θ, cos 2θ, or tan 2θ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of θ in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]		

7 **(i)** Use a correct method to express $\overrightarrow{OP}$ in terms of λ M1

Obtain the given answer A1 [2]

(ii) *EITHER*: Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$ in terms of λ M1

Using the correct method for the moduli, divide scalar products by products of moduli and express cos AOP = cos BOP in terms of λ, or in terms of λ and OP M1*

OR1: Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of λ M1

Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express cos AOP = cos BOP in terms of λ, or λ and OP M1*

Obtain a correct equation in any form, e.g. $\dfrac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}} = \dfrac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$ A1

Solve for λ M1(dep*)

Obtain $\lambda = \frac{3}{8}$ A1 [5]

[SR: The M1* can also be earned by equating cos AOP or cos BOP to a sound attempt at cos $\frac{1}{2}$ AOB and obtaining an equation in λ. The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.]

[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for OP to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such cases.]

(iii) Verify the given statement correctly B1 [1]

8 **(i)** Use any relevant method to determine a constant M1

Obtain one of the values $A = 3$, $B = 4$, $C = 0$ A1

Obtain a second value A1

Obtain the third value A1 [4]

(ii) Integrate and obtain term $-3\ln(2-x)$ B1√

Integrate and obtain term $k\ln(4+x^2)$ M1

Obtain term $2\ln(4+x^2)$ A1√

Substitute correct limits correctly in a complete integral of the form $a\ln(2-x) + b\ln(4+x^2)$, $ab \neq 0$ M1

Obtain given answer following full and correct working A1 [5]

9	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = e^{-\frac{1}{2}}$, or equivalent	A1	
		Obtain answer $y = -\frac{1}{2}e^{-1}$, or equivalent	A1	[5]
	(ii)	Attempt integration by parts reaching $kx^3 \ln x \pm k \int x^3.\frac{1}{x}\,dx$	M1*	
		Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,dx$, or equivalent	A1	
		Integrate again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent	A1	
		Use limits $x = 1$ and $x = e$, having integrated twice	M1(dep*)	
		Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent	A1	[5]
		[SR: An attempt reaching $ax^2(x \ln x - x) + b\int 2x(x \ln x - x)\,dx$ scores M1. Then give the first A1 for $I = x^2(x \ln x - x) - 2I + \int 2x^2\,dx$, or equivalent.]		

10	**(a)**	*EITHER*:	Square $x + iy$ and equate real and imaginary parts to 1 and $-2\sqrt{6}$ respectively	M1*	
			Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$	A1	
			Eliminate one variable and find an equation in the other	M1(dep*)	
			Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent	A1	
			Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$	A1	[5]
		OR:	Denoting $1 - 2\sqrt{6}i$ by $R\text{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\text{cis}(\frac{1}{2}\theta)$ and find values of R and either $\cos\theta$ or $\sin\theta$ or $\tan\theta$	M1*	
			Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta)$, and $\cos\theta = \frac{1}{5}$ or $\sin\theta = -\frac{2\sqrt{6}}{5}$ or $\tan\theta = -2\sqrt{6}$	A1	
			Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$	M1(dep*)	
			Obtain $\cos\frac{1}{2}\theta = \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta = \pm\sqrt{\frac{2}{5}}$, or equivalent	A1	
			Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$, or equivalent	A1	
			[Condone omission of ± except in the final answers.]		

	(b)	Show point representing 3i on a sketch of an Argand diagram	B1	
		Show a circle with centre at the point representing 3i and radius 2	B1√	
		Shade the interior of the circle	B1√	
		Carry out a complete method for finding the greatest value of arg z	M1	
		Obtain answer 131.8° or 2.30 (or 2.3) radians	A1	[5]
		[The f.t. is on solutions where the centre is at the point representing –3i.]		